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ON PROBABILITY IN GEOTECHNICS

RANDOM CALCULATION MODELS EXEMPLIFIED ON SLOPE STABILITY ANALYSIS AND GROUND-SUPERSTRUCTURE INTERACTION

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SUMMARY

"The problems of soil mechanics may be divided into two principal groups - the stability problems and the elasticity problems. The stability problems deal with the conditions for the equilibrium of ideal soils immediately preceding ultimate failure by plastic flow."

"Elasticity problems deal with the deformations of the soil due to its own weight or due to external forces such as the weight of the building. All settlement problems belong in this category." (Terzaghi, 1943)

Σ.1 General

In this thesis different aspects of geotechnical modelling are discussed. It is a combination of elements from three different academic disciplines:

- Geotechnics
- Structural mechanics
- Statistics

The work is based upon examples from two fields of geotechnical modelling:

- Slope stability, i.e. an example of stability problems
- Interaction ground/superstructure, i.e. an example of elasticity problems

All sorts of technical calculations involve a certain amount of uncertainty. In this thesis a probabilistic approach is used as a way to quantify uncertainty.

New calculation methods should, to be operative in design, show a balance between:

- Realistic description of reality
- Simplicity
- Improvement of the state-of-the-art
- Recognition.
- Adaptation to codes
- Possibility of further improvements

Hence improved accuracy and precision of a method is not necessarily an improvement in engineering practice. In design the level of refinement has to be valued in the light of the purpose of the method. In the author's experience three different levels of complexity is a useful separation of methods with different accuracy and precision.

Σ.2 Calculation models

During the last decades it has become mandatory to verify structures in two different limit states, the ultimate limit state and the serviceability limit state. Different calculation models can be used for the verification of the two different limit states.

Safety margin

In the evaluation of safety, the concept factor of safety, $F = R/S$, has traditionally been used in both geotechnical and structural engineering. A complication, when engineers from the two disciplines have to co-operate is that the geotechnical engineer often considers uncertainty as a resistance problem, while the structural engineer is more concerned with the uncertainty of the actions. Hence it is not indisputable whether an improvement should be regarded as an increase of the resistance or a decrease of the action. The alternative safety concept, the safety margin $M = R - S$, does not have this draw-back. In this thesis a dimensionless safety margin is presented as an alternative:

$$m = \frac{R-S}{R} \quad \text{(i)}$$

with the convenient range [0-100%]. There is a unique relation between this dimensionless safety margin and the factor of safety, $m = 1-(1/F)$. This means that the safety margin, in the same way as the factor of safety but unlike the conventionally defined safety margin, can be used as a relative measure of safety, e.g. to determine the critical slip surface in a slope stability analysis.

Probabilistic interpretation of deterministic modelling

In traditional, deterministic modelling the action and the resistance are represented by fixed and known values. Such an interpretation implies that the safety margin actually has a sufficiently large, positive value.

In a probabilistic approach both the resistance and the action can take a wide range of values. This can be interpreted as that the action and the resistance have fixed but unknown values. The same is then valid for the safety margin. A sufficiently low probability of failure is obtained if the safety margin is at least not negative for a physically possible but unlikely combination of the resistance and the action.

Random models to describe uncertainty

To describe the unlikeness mentioned in the previous section, variables in the calculation are given as random variables, i.e. variables in which the probability of different outcomes is given by probability distributions. A number of distributions can be used for this purpose, for example:

- normal distribution
- lognormal distribution
- Extreme value distributions
- β-distributions
- Exponential distribution

The choice of distribution for a particular random variable is far from trivial from a statistical point of view. This is valid especially for calculations in the ultimate limit state in which very low values of the probability of failure are foreseen. Hence the tails of the distributions govern the design. An ordinary geotechnical testing then represents too small samples for the evaluation of an underlying distribution. In principle the type of distribution should be chosen from the physical characteristics of a variable. However, in the applications in this thesis the probability of failure represents a formal probability, at least in the ultimate limit state. In such cases the choice of distribution can often be seen as a code issue, i.e. the probability demanded and the choice of distribution is an inseparable pair. In applications of this latter type the lognormal distribution can serve as a comprehensive distribution, at least in a probabilistic analysis of the main characteristics of a problem. The normal distribution, which is defined for negative values, has to be used with caution as the negative values often represent physically impossible situations.

Algorithms

The physical model in a probabilistic analysis does not have to differ from the model used in a deterministic analysis. However, for the mathematical solution one has to use special algorithms:

- Mathematical analysis
- PEM
- Monte-Carlo simulation
- Reliability analysis

The algorithms presented above serve as useful tools in the solution of probabilistic analyses. The mathematical analysis is a straight forward method, however, restricted to not too complex problems. In *PEM*, the point estimate method, the input variables are given as two-point estimations, see Figure Σ.i. It is a simple method, which is easy to apply in a traditional deterministic model. The results are restricted to approximate values of the mean value and the variance of unknown variables. The practical application is limited to problems with a small number of random variables.

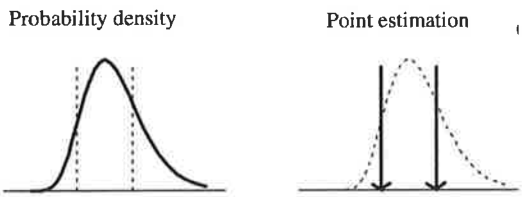


Figure Σ.*i* Principle of point estimate method - PEM

In a Monte-Carlo simulation values of input variables are simulated from a given distribution of the variable, see Figure Σ.ii. The simulation results in the complete distribution of any unknown, resulting variable. The accuracy is governed by the number of iterations. Only practical reasons limit the accuracy of the results. To obtain the tails of a distribution a very large number of iterations is required.

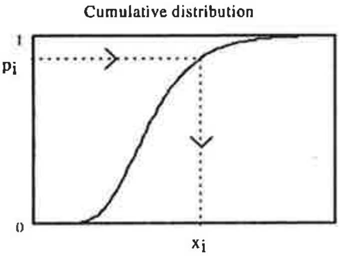


Figure Σ.ii Iteration step of a basic variable in Monte-Carlo simulation

In reliability analysis, the probability of failure can be represented by the reliability index β, see Figure Σ.iii.

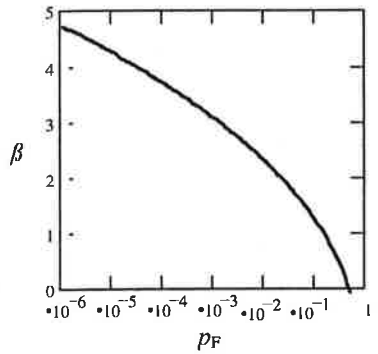
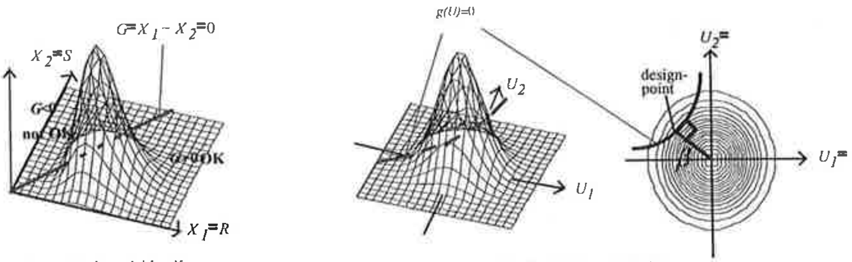


Figure Σ.iii Relation between reliability index β and formal probability of failure p_F.

Figure Σ.iii is based upon the original definition of β as a function of the parameters of the safety margin:

$$\beta = \frac{\mu_M}{\sigma_M} \qquad (\Sigma.ii)$$

To obtain a formulation, which is invariant of the mathematical formulation of the safety margin the reliability index can be defined as a geometrical property related to the failure surface, the reliability index $\beta_{H\text{-}L}$. The basic random variables X of a problem then have to be transformed into a set of independent standard normal variables U, cf. Figure Σ.iv.



x-space. Basic variables X, Limit state function $G(X)$

u-space: Standard normal distributions Transformed limit state function $g(U)$

Figure Σ.iv Transformation of basic variables from x-space to u-space. The reliability index $\beta_{H\text{-}L}$.

Reliability analysis is used to calculate very small probabilities, preferably probabilities of failure. In this way the reliability analysis and the Monte Carlo simulation complement each other

Σ.3 Soil properties

When describing soil properties as random variables different uncertainties have to be accounted for

- Natural variation in the soil
- Systematic errors in the test method
- Random errors due to the test method
- Errors due to limited number of tests

Geostatistics

Geotechnical testing gives test values for soil properties in single points, while geotechnical problems usually are governed by the average values of soil properties of a volume. In geostatistics soil properties are modelled as random fields, i.e. an infinite number of random variables, each applied in a single point of a soil volume. The main purpose of geostatistics is to describe the dependence between these variables. Figure Σ.v shows observations of the shear strength versus depth together with two different alternative of a trend model of the shear strentgth. Deviations from the trend are in geostatistics considered as random variables. Plots of the correlation between such pairs of random variables, as a function of the distances between them, are called correlogram.

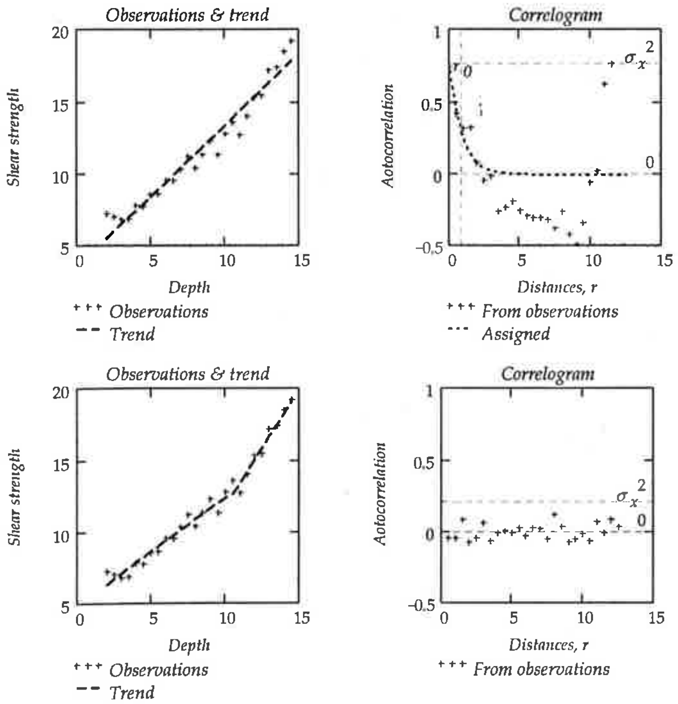


Figure Σ.v Influence on a statistical analysis from alternative models of reality.

Figure Σ.v draws the attention to an important issue in probabilistic modelling. The correlograms in the figure give completely different results for two, from a deterministic point of view, similar models of the shear strength. Hence, in random modelling the variations and the applied model form an unseparable pair. If both the test method and the model are perfect no variations exist. An interpretation of this is that variations in a probabilistic analysis are a measure of lack of knowledge.

Bayesian statistics

In traditional geotechnical design, pre-knowledge and test results are combined. In a statistical analysis this can be done systematically by Bayesian statistics. As an example, a prior distribution of an unknown mean value of a basic variable can be updated based upon added information, e.g. test results, see equation Σ.iii.

$$\begin{pmatrix} \textit{Posterior prob.} \\ \textit{of the true mean} \\ \textit{given the sample} \end{pmatrix} = \begin{pmatrix} \textit{Norma-} \\ \textit{lising} \\ \textit{constant} \end{pmatrix} \cdot \begin{pmatrix} \textit{Sample likelihood} \\ \textit{given} \\ \textit{the true mean} \end{pmatrix} \begin{pmatrix} \textit{Prior prob.} \\ \textit{of the true} \\ \textit{mean} \end{pmatrix} \qquad (\Sigma.\text{iii})$$

Thus with the added information the uncertainty of the unknown parameter is reduced.

Assume a situation in which pre-knowledge shall be valued as equivalent to the results of a test series. The pre-knowledge of the property can then be interpreted as an equivalent sample $\mathbf{x}_1$ of m tests such that:

$$\mu' = \bar{x}_1 \text{ and } m = \frac{\sigma^2}{\sigma'^2} \qquad (\Sigma.\text{iv})$$

if the standard deviation is known and

$$\ln \mu' = \overline{\ln x_1} \text{ and } m = \frac{V^2}{V'^2} \qquad (\Sigma.\text{v})$$

if the coefficient of variation is known. The parameter σ or V here denotes uncertainty of the basic variable, i.e. including measurement errors, while σ′ or V' denote the uncertainty of the mean value of the basic variable. The result of such an up-dating process is given in Table Σ.ι.

	Prior information		**Up-dating no.1**		**Up-dating no. 2**	
	μ [kPa]	V [%]	μ [kPa]	V [%]	μ [kPa]	V [%]
Prior	-	-	15,6	19	14,1	12
Likelihood	-	-	13,9	6	13,8	5
Posterior	-	-	14,1	6	14,0	4
Baysian	15,6	22	14,1	12	14,0	11

Table Σ.i Baysian up-dating of the undrained shear strength. (=table 3.1 for z=10)

Pore pressure

It is a well-known fact that many failures in geotechnical design originate in mistakes in the assessment of the pore pressure. Hence, a procedure of a probabilistic description is of interest. An investigation of ground-water data from 39 observation series from the south of Sweden, shows that the application of a 'best' choice of distribution is far from trivial, see Figure Σ.vi. As could be expected, variations in the topography of the landscape play a more important role than the choice of probability distribution.

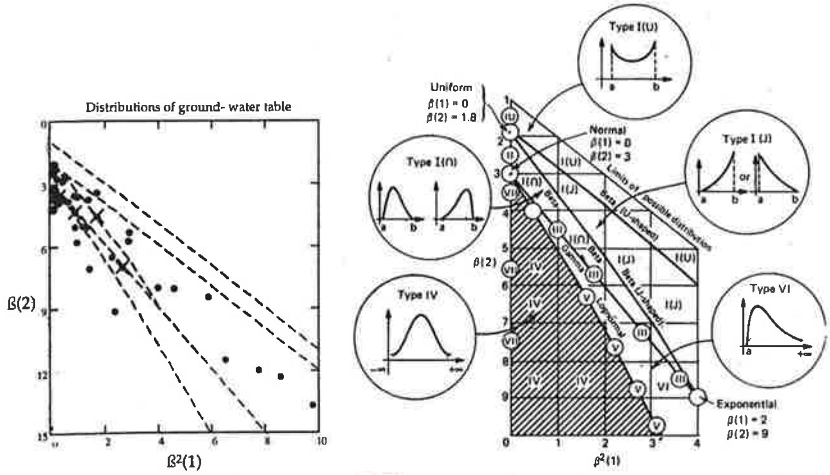


Figure Σ.vi Distributions of groundwater observations. Annual maxima.

Shear strength

The drained shear strength is normally given as a function of two effective stress parameters, the cohesion intercept c' and the friction angle ϕ. The difficult problem to asses the correlation between these two variables can be eliminated from the analysis by treating the shear strength, in a stress range of interest, as a random variable. The shear strength can in many practical cases be modelled in a wide stress range with only $\tan\phi$ as a random variable. This requires the use of the concept of attraction, i.e. cohesion seen as pre-stress instead of as adhesion.

If one wants to describe the undrained shear strength with effective stress parameters it s necessary to know the pore pressure in a failure zone is known. The magnitude of shear induced pore pressure is a complex problem. Hence in practical cases one is often forced to determine the shear strength as an 'index' value from field or laboratory tests. Hence the uncertainty of the test method has to be incorporated into a probabilistic description.

Σ.4 Slope stability - Ultimate limit state design

A pragmatic way to describe different situations of drainage conditions in a slope is to separate slope stability analysis into:

- Drained analysis
- Undrained analysis
- Combined analysis

Shear strength

To determine the in-situ stress state in a slope is difficult. Despite this, great efforts are made in slope stability analysis to relate the shear strength to this 'unkown' stress state:

$$c = c' + (\sigma - u) \cdot \tan(\phi) \qquad (\Sigma.\text{vi})$$

In this thesis an alternative proposal is presented, i.e. to define the shear strength as the shear stress at failure:

$$c = \tau_{ult} = c' + (\sigma_{N,ult} - u) \cdot \tan(\phi) \qquad (\Sigma.\text{vii})$$

in which $\tau_u \neq F \cdot \tau$. Besides the logical advantage with the definition, the amount of calculation work is reduced considerably in a slope stability analysis. This latter advantage is the main purpose of the definition in this thesis.

'Stable' slopes

An often raised issue is how to interpret in a probabilistic way the fact that a natural slope has proved to be stable during a long time. In this thesis the idea is presented that the observed fact of stability can be regarded as an ultimate control, hence the probability distribution for the safety margin can be truncated. This means that a slope with a long history has a larger safety than a new slope if the slopes in all other respects are equivalent.

Application

In the beginning of this section it was stated that a useful separation of calculation models is in three levels of complexity. Such a separation for slope stability analysis is summarised in the following.

Level 1- 'Design chart'

The level 1 method presented in this thesis is a probabilistic interpretation of the well-known 'design chart' method given by the equation:

$$F = \frac{N \cdot \overline{c_u}}{P_d} \qquad (\Sigma.\text{viii})$$

with the safety margin $m = \ln(F)$. As an example, the distribution of the safety margin is given in Figure Σ.vii. The scale of the y-axis is in a normal probabilistic scale, i.e. a normal distribution gives a straight line. The simplicity of the procedure makes it a good complement to a more careful, deterministic analysis as well as a starting point for a comprehensive probabilistic analysis.

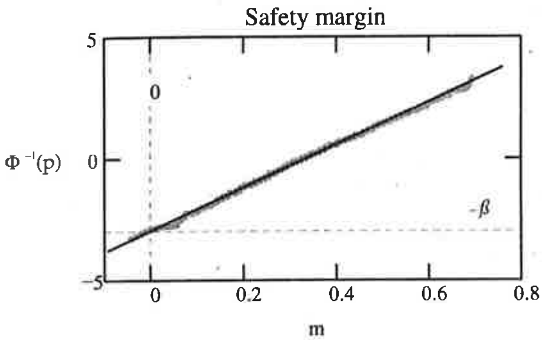


Figure Σ.vii Level 1. Undrained analysis. Reliability analysis and Monte Carlo simulation of the safety margin m=ln(F).

Level 2 - Bishop's simplified method

Bishop's simplified method is a well-known and often used model for slope stability analysis. In this thesis the method is combined with the shear strength presented in the section 'shear strength' and adapted to combined analysis. A result obtained from the analysis is that different types of analyses result in different critical slip circles, see Figure Σ.viii. The distribution of the safety margin for the 'most' critical slip circle is shown in Figure Σ.ix.

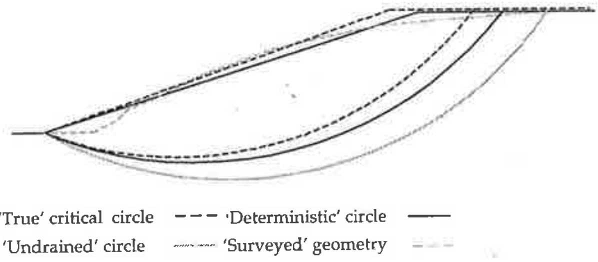


Figure Σ.viii 'Critical' slip circles

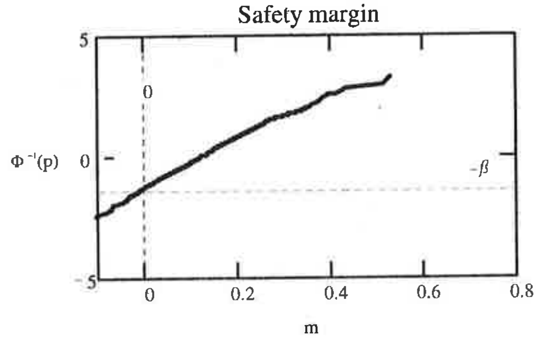


Figure Σ.ix Level 2. Combined analysis. Monte Carlo simulation of the safety margin. Alt. 'true critical circle'

Level 3 - A shear beam model

A proposal for a discrete element model for slope stability analyses is outlined. A slope is in the method modelled as a frame work of shear beams, see Figure Σ.x.

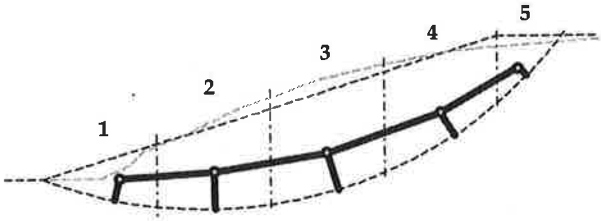


Figure Σ.x Geometry of the shear beam model.

The method does not include any assumption of a constant degree of mobilisation along a slip surface. Instead the deformation properties of the soil are considered. Figure Σ.xi shows the distribution of normal stresses at the slip surface for two different assumptions of stiffness distribution.

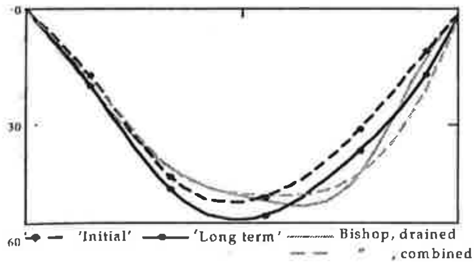


Figure Σ.xi Normal effective stress distribution.

The result of a probabilistic analysis is shown in Figure Σ.xii. The figure is an analysis of the same slip circle as the result in Figure Σ.ix.

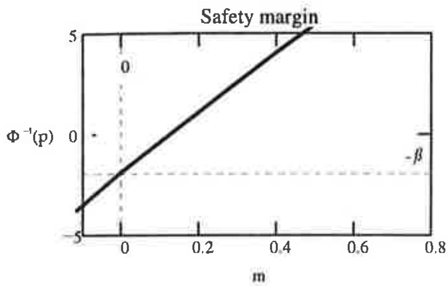


Figure Σ.xii Level 3. Shear beam model. Combined analysis. PEM-calculation of the safety margin. Alt. 'true critical circle'

Σ.5 Interaction ground/superstructure

Interaction between the ground and the superstructure is an issue of combining geotechnical and structural engineering. This thesis presents a number of means aimed at this purpose.

Spread foundations

As a tool of rapid assessments of stresses and strains in the soil Boussinesque's solution for an elastic half sphere is given an approximate formulation with high accuracy. Based upon this the vertical stress for a uniformly distributed surcharge p on an area B times L becomes:

$$\sigma_v(x,y,z) = \frac{p}{\pi^2} \cdot \left[\left[\frac{2z(B+2x)}{4z^2+(B+2x)^2} + \text{atan}\left(\frac{B+2x}{2z}\right) \right] + \left[\frac{2z(B-2x)}{4z^2+(B-2x)^2} + \text{atan}\left(\frac{B-2x}{2z}\right) \right] \right] \cdot \left[\left[\frac{2z(L+2y)}{4z^2+(L+2y)^2} + \text{atan}\left(\frac{L+2y}{2z}\right) \right] + \left[\frac{2z(L-2y)}{4z^2+(L-2y)^2} + \text{atan}\left(\frac{L-2y}{2z}\right) \right] \right] \qquad (\Sigma.\text{ix})$$

Closed formulas for the settlements of the ground surface can be obtained by integration. As an example , the settlements of a line load can be approximated as:

$$s(x) \approx \frac{P}{\pi \cdot M} \cdot \begin{cases} \left[\left(\frac{50 \cdot H}{40x+H} \right)^{2/3} - 1 \right] & ; \; x > B/4 \\ \left[\left(\frac{50 \cdot H}{10 \cdot B+H} \right)^{2/3} - 1 \right] & ; \; x < B/4 \end{cases} \qquad (\Sigma.\text{x})$$

Piled foundation

Piled foundation adds an extra complexity as two elements with very different stiffness, the pile element and the soil have to interact. A useful tool is the concept 'the neutral plane', which can be estimated as:

$$z_{Nf} \approx 0{,}7 \cdot (1-\eta) \cdot L \qquad (\Sigma.\text{xi})$$

for constant shaft friction and as:

$$z_{Ng} = 0{,}85 \cdot \sqrt{1-\eta} \cdot L \qquad (\Sigma.\text{xii})$$

for increasing shaft friction, where in both formulas η is the degree of mobilisation for the shaft friction.

A creep model

To determine volumetric creep deformations in clay a simple rheological model is given, see Figure Σ.xiii. Based upon the model the total deformations can be determined as a sum of elastic/plastic deformations and creep deformations by the equation:

$$s(t) = U_{\sigma\tau}(t) \cdot \frac{P}{S} + s_{\sigma\tau}(t) \qquad (\Sigma.\text{xiii})$$

The elastic/platic deformations are delayed compared to classical theory, Figure Σ.xiv.

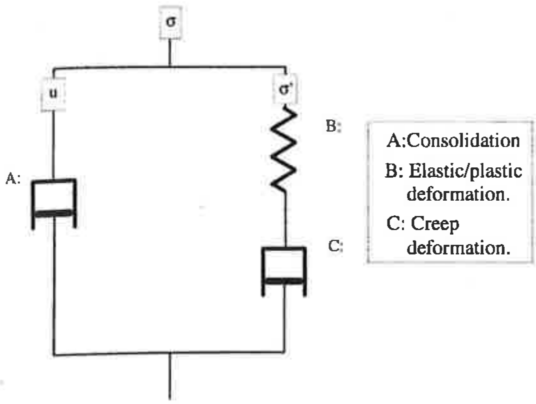


Figure Σ.xiii Rheological creep model

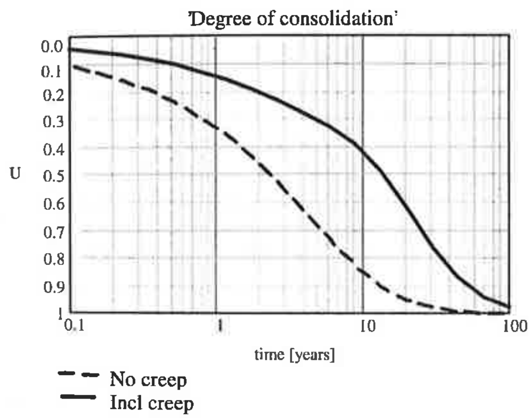


Figure Σ.xiv Influence of the dissipation of excess pore pressure due to creep.

An example of an application of the creep model outlined above is presented in Figure Σ.xv. The results are compared with results from a finite element method developed in Trondheim.

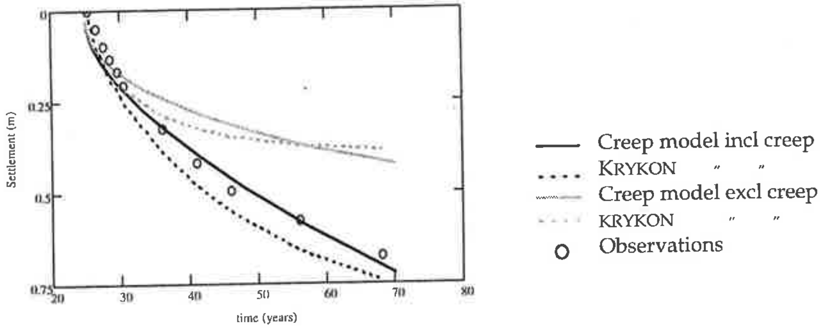


Figure Σ.xv Oslo railroad customs building. Calculated and observed settlements. From (Alén, 1998b) and (Svanø et al., 1991).

Application

Similarly as for slope stability applications are presented for three different levels of refinement. Figure Σ.xvi shows the cross-section of the analysed superstructure.

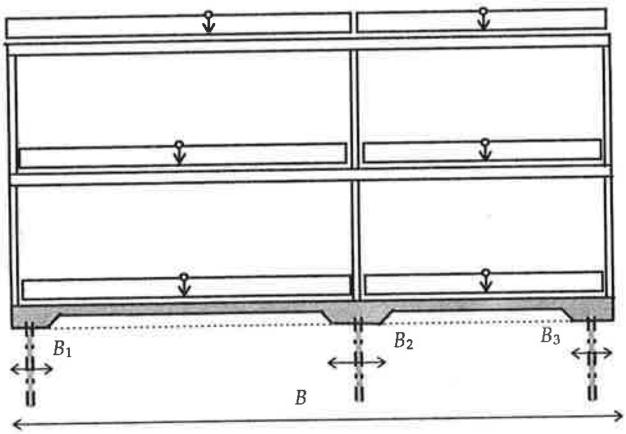


Figure Σ.xvi Cross-section of superstructure

Level 1 is based upon assumptions of rigid superstructure or fixed supports alternatively, Figure Σ.xvii.

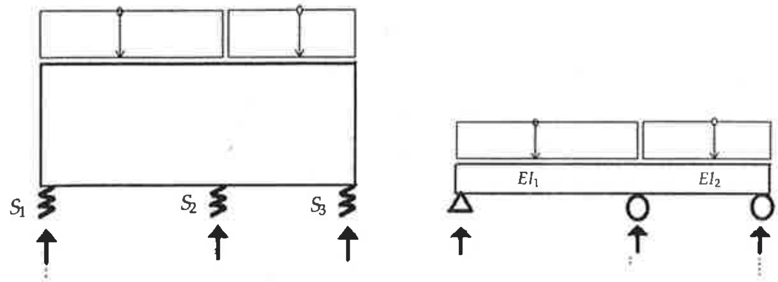


Figure Σ.xvii Level-1. Interaction model:
Rigid superstructure/Flexible supports and
Fixed supports/Flexible superstructure respectively.

Level 2 is an analysis of an elastic superstructure and spring supports, Figure Σ.xviii.

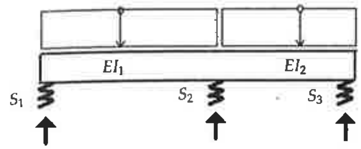


Figure Σ.xviii Level-2. Interaction model:
Flexible superstructure/Flexible supports.

In level 3 interaction in the ground between the supports is considered.

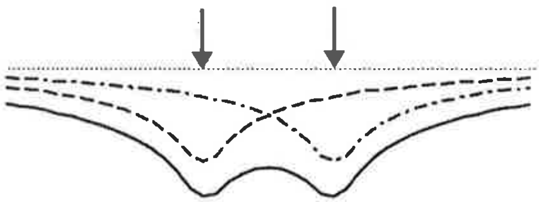


Figure Σ.xix Interaction in the ground between supports

A soil beam model represents the soil in the interaction analysis, Figure Σ.xx. The soil beam is a continuous shear beam on elastic supports. The idea behind the model is to calibrate the properties of the beam against a more rigorous geotechnical model.

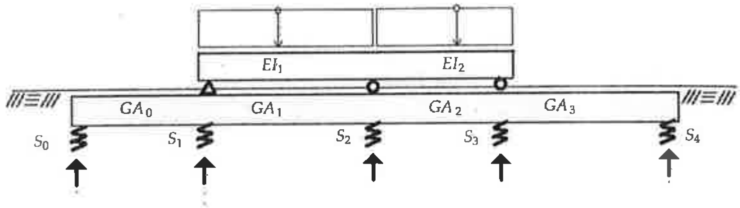


Figure Σ.xx Level-3. Soil beam model. Interaction between supports. Flexible superstructure/Flexible supports.

Figure Σ.xxi shows settlements obtained with the soil beam model.

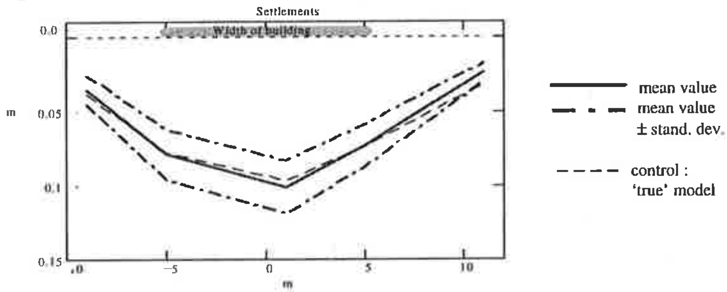


Figure Σ.xxi Soil beam model. Settlements.

Reactions and a bending section moment in the superstructure is summarised in Table Σ.ii. The table shows clearly the difficulties to determine appropriate section forces in a superstructure.

	R_1 [kN]		R_2 [kN]		R_3 [kN]		M [kNm]	
	μ	V%	μ	V%	μ	V%	μ	V%
Level 1	*Rigid superstructure*							
	49	22	130	15	22	35	-70	17
Level 1	*Fixed supports*							
	73	20	66	19	61	18	80	28
Level - 2	*Flexible superstructure*							
	65	20	88	17	47	21	28	60
Level - 3,	*Soil beam model*							
	65	20	86	15	48	19	32	38

Table Σ.ii Example of interaction models Reaction and section forces.